Reply to the comment on 'Correlative amplitude–operational phase entanglement embodied by the EPR-pair eigenstate $|\eta\rangle$'

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REPLY

Reply to the comment on 'Correlative amplitude–operational phase entanglement embodied by the EPR-pair eigenstate $|\eta\rangle$'

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Abstract
We compare and contrast our amplitude–phase entanglement with that of Luis in his comment. Luis's entangled state is defined in a finite Fock space. His comment on the operational phase operator seems to be contradicting the original meaning of Mandel *et al*.

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It is worth comparing the two approaches for discussing amplitude (number)–phase entanglement in [1, 2]. In [2] we pointed out that the common eigenstate $|\eta\rangle$ of two particles' relative coordinate and total momentum not only involves the original Einstein–Podolsky–Roson coordinate–momentum entanglement, but also embodies the correlative amplitude–operational phase entanglement. The operational phase operator $e^{i\Phi} = \sqrt{\left(a_1 - a_2^\dagger\right)/\left(a_1^\dagger - a_2\right)}$ [3–5] is used by Mandel *et al* [4] in an eight-port balanced homodyne detector experiment to represent the measurement of phase difference between the local oscillator field and a quantum field. As Mandel *et al* emphasized in [6]: 'As measurements always involve the difference between two phases, and as an interference or homodyne experiment usually yields the cosine or sine of the phase difference', they introduced measured operators for the cosine and sine of the phase difference (see also [7, 8]). Based on $e^{i\Phi}$ we addressed the correlative amplitude–operational phase entanglement in [2]. Thus the comment in [1] that '. . . the author considers another operator that no longer represents the genuine phase difference variable, but a noise version of a single mode phase' contradicts a series of references [3–8] in this respect.

Instead of using $e^{i\Phi}$, Luis *et al* [9] studied their phase-difference operator by examining the polar decomposition of $a_1 a_2^\dagger = E_{12}\sqrt{a_1^\dagger a_1\left(a_2^\dagger a_2 + 1\right)}$. In the subspace spanned by the Schwinger angular momentum state $|j, m\rangle$, denoted by a superscript (N), $N = 2j$, the phase-difference operator is $E_{12}^{(N)} = \sum_{m=-j+1}^{j} |j, m-1\rangle\langle j, m| + e^{i(N+1)\varphi_0}|j, j\rangle\langle j, -j|$. Thus Luis–Sánchez-Soto's angle operator and the amplitude–phase entangled states are defined in a finite space. For describing a Hermitian rotation angle, Pegg and Barnett [10] have invented an approach which removes the indeterminacies caused by working directly with an infinite-dimensional state space.

In sum, Luis's approach, which is valuable and important, is defined only in a finite Fock space. If one needs the number–phase difference entangled state in an infinite Fock space, one should use the operational phase operator of Mandel *et al*.

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